Mass Transfer Coefficients for Solids Suspended in Agitated Liquids

JAMES J. BARKER and ROBERT E. TREYBAL

New York University, New York, New York

The mass transfer coefficient in covered, right-cylindrical tanks full of liquid, turbulently agitated at various speeds by turbines with six flat blades, was measured by the rate of solution of suspended solids in water and in 45% sucrose solutions.

Screened crystals in the following U. S. mesh sizes were used: boric acid: 18/20, 16/18, 16/20, 14/16, 12/14, 10/12, 8/10, 6/8; rock salt: 6/8, 4/6. Pellets were benzoic acid: 0.126 in. long by 0.218-in. diam.; salt: 0.565-in. diam. by 0.531-in. long (over rounded ends). Tanks were 6, 12, 18, and 30 in. Turbines were 2, 3, 4, 6, 9, and 12 in. in diameter, centrally located. Four full-length baffles 10% of the tank diameter wide were spaced at 90 deg. A few runs were made without baffles.

The coefficient of mass transfer was found to be independent of particle size and Schmidt number (N_{Sc} = 735 to 62,000) and could be correlated with turbine Reynolds number in each tank, with larger tanks yielding smaller coefficients at the same N_{Re}. An empirical equation which fits all the data from the baffled tanks within about 4% (in the range $0.1 < k < 2$) is

$$\ln (10\,k) = I_2 + 0.85\,V^{0.02975} \ln (N_{Re}/10^4)$$

where $I_2 = 0.8235 - 1.544\,V^{1/3} + 0.115\,V^{2/3}$

The variance of estimate for this expression is 0.0383, in units of $[\ln(10\,k)]^2$.

For extrapolation outside the experimental range of vessel sizes it is recommended that $I_1 = 0.676 - 1.266\,V^{1/3}$ be used in place of I_2. $N_{Re} = \pi\,T^2n/\nu$. The results indicate that power per unit volume for a given k goes through a maximum, with the following relative values for the 6-, 12-, 18- and 30-in. tanks: 1, 1.73, 1.78, 0.62.

A treatment of the data according to dimensionless groups provides another correlation:

$$kd/D = 0.02\,N_{\mathrm{Re}}^{0.833}\,N_{Sc}^{0.5}$$

t is shown that for the systems used $1/D$ is essentially proportional to $N_{Sc}^{0.5}$, and so the effect of diffusivity here is only apparent.

A recent review of the available information for computing the stage efficiency of mixer-settler extractors (*19*) revealed the severe limitations of our knowledge of the mass transfer coefficients for the continuous phase which surrounds the dispersed particles in the mixer. For baffled vessels it was even necessary to resort to the use of coefficients for the continuous phase in contact with the surface of the vessel itself. This paper attempts to fill this gap by studying the rate of dissolution of particulate solids in agitated liquids under circumstances such that the surface area is known.

In 1897 Noyes and Whitney (*15*) formulated a durable theory for the dissolution of solids in liquids, which is summarized by

$$N = k\,(c_s - c) \qquad (1)$$

Since then a number of investigators have attempted to correlate k by dimensional analysis with the apparent variables, such as particle size, turbine parameters, vessel geometry, fluid properties, density difference between solid and liquid, and ratio of the phases.

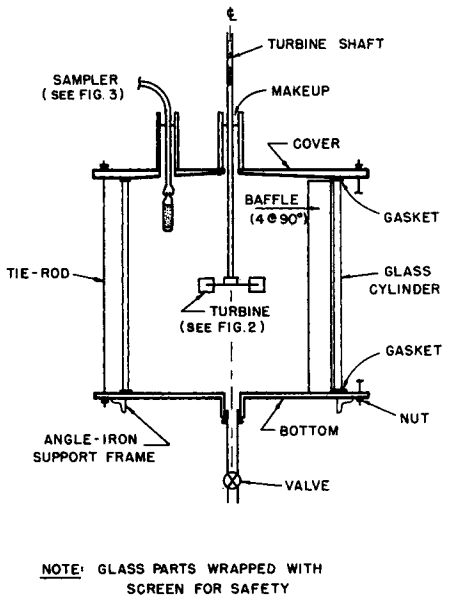


Fig. 1. Typical sketch for 6-, 12-, and 18-in. vessels. (See Table 1 for dimensions, etc.)

Murphree (*13*), Hixson and coworkers (*3, 4, 5*), and White (*20*) assumed no effect of particle size, but others (*1, 21, 8, 16, 7, 9, 18*) either found or assumed an effect.

A great variety of impeller types have been investigated, but relatively little information is available on flat-blade turbines, and no systematic investigation of the effect of ratio of turbine diameter to tank diameter has been made. The effect of speed for fully suspended particles has ranged from n^0 (*1, 8, 12, 20*) to $n^{0.9}$ (*6*), as well as intermediate effects (*3, 11, 16, 12, 14, 7*).

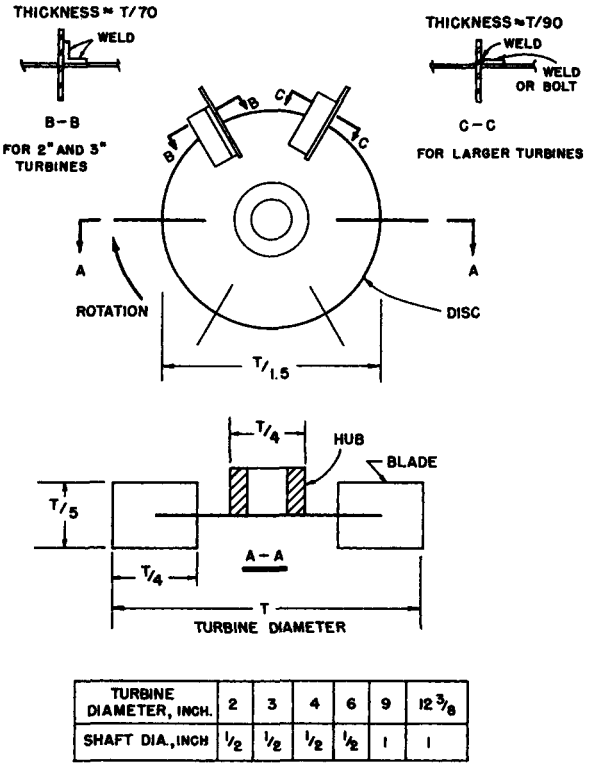
Essentially all the work has been done in unbaffled vessels with an air-liquid interface. Mack and Marriner (*11*) and others (*8, 20, 14, 16, 12, 6, 7*) worked with baffled vessels, but the effect of baffles is not established; neither is the shape of the vessel.

The significant fluid properties are kinematic viscosity, which in the usual dimensional-analysis treatment enters into Reynolds numbers and Schmidt number, and diffusivity, which is included in the latter and the Sherwood number. The effect of kinematic viscosity in the Reynolds number is usually the same as that of speed, which has been mentioned above. Hixson and Baum additionally related (kd/D) to $(\nu/D)^{0.5}$, but the exponent 0.5 was suggested by the results of gas-liquid contact in wetted-wall towers rather than by their own data. Others (*8, 16, 6*) have assumed the same relationship, but Aksel'rud (*1*) changes this to $(\nu/D)^{1/3}$. No systematic study of the effect has been made.

Most investigators assumed no effect of density difference. Aksel'rud (*1*) suggests $k \sim [(\Delta\rho)/\rho]^{1/4}$ and Kneule (*8*) $[(\Delta\rho)/\rho]^{1/8}$, both of which lead

TABLE 1. VESSEL DIMENSIONS AND MATERIALS

Nominal size of vessel, in.	6	12	18	30
Height, inside wall, in.	5.95	12	18.1	30
Diameter, inside, in.	5.45	11.2	17.4	30
Volume, inside, cu. ft.	0.0806	0.685	2.481	12.22
Wall material	Pyrex	Pyrex	Pyrex	316 stainless steel
Baffle width, in.	0.55	1.12	1.74	3
thickness, in.	1/16	1/16	1/16	1/4



TURBINE DIAMETER, INCH.	2	3	4	6	9	12 3/8
SHAFT DIA., INCH	1/2	1/2	1/2	1/2	1	1

Fig. 2. Sketch of turbines.

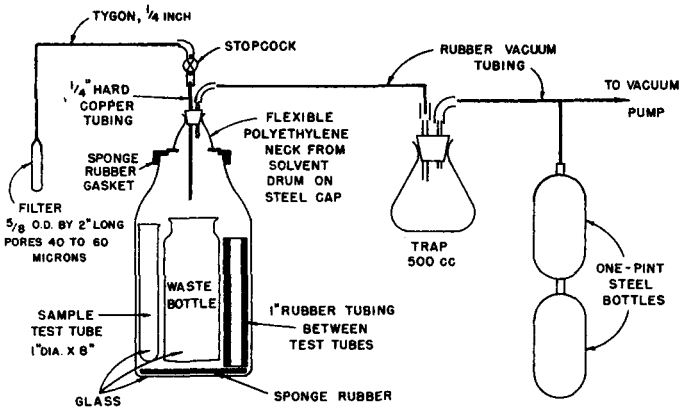


Fig. 3. Sampling system.

to unreasonable values of k as $\Delta\rho$ approaches zero. Kolář (9) suggests $(\rho_s/\rho)^a$.

Most of the work has been done with dilute suspensions, with no attempt to study phase ratio. Wilhelm (21) however finds k decreasing with decreasing percentage of solids.

The effects of the variables on k are clearly not well established.

APPARATUS AND PROCEDURE

Figure 1 and Table 1 give the design and dimensions of the vessels used; Figure 2 summarizes the turbine dimensions.

Carefully screened fractions of boric acid and rock salt, as well as pellets of salt and of benzoic acid, were dissolved in water or 45% sucrose solutions in water under the circumstances outlined in Table 2. Table 3 summarizes Schmidt numbers and density differences.

Following is an outline of the procedure for each run. Fill the vessel and make-up reservoir with liquid of the same concentration. Measure the liquid temperature and adjust the turbine speed to proper range. Stop the turbine, remove a portion of liquid, add weighed solids, and replace the liquid. Quickly insert the sampling filter, start the turbine, and, as soon as solids are fully suspended—almost immediately with small amounts of solids, and within a few seconds in any usable instance—start the stop watch. Ten or fifteen seconds later draw the first sample as follows (refer to Figure 3). Open the stopcock on the sampler 1 sec. before the recorded time, directing the flow into a waste bottle. At recorded time divert the flow into the proper test tube by maneuvering the hard copper tubing extending through the flexible polyethylene neck on the cover of the vacuum bottle; about 40 ml. or 1 sec. later, whichever is less, stop the flow by shutting the stopcock. If the sample volume is not replaced automatically from the reservoir, add the make-up. (This applies only to early runs on the 12-in. vessel.) Draw the second sample after the interval listed in the last column of Table 2, and the third sample a similar interval after that. For example in the first run indicated in Table 2, samples were drawn 10, 35, and 60 sec. after the particles were suspended. Read the turbine speed by a tachometer and the temperature by a thermometer between and/or after samples. Withdraw the filter and place it in distilled water, read the temperature, stop the turbine, drain and clean the vessel. Meanwhile break the vacuum and remove the sample bottle. Rinse and dry the filter with an auxiliary bottle. Pipette two or three 10-ml. portions from each sample and titrate; rinse and dry the pipette in an oven between samples. Measure the remaining volumes of samples in test tubes and the volume of liquid in the waste bottle. A total of 128 such runs were made. In most of the runs additional samples were drawn at intervals after the first. Only the data from the first interval, which were the more reliable, are reported here.

Experimental conditions were chosen so that only a small fraction of the initial solids was dissolved during a run. This permitted estimation of the interfacial area and consequently allowed calculation of true mass transfer coefficients from the following simple equation:

$$\ln\left(\frac{c_s - c_i}{c_s - c}\right) = \frac{k\overline{A}}{V}(t - t_i) \qquad (2)$$

Since the material removed from the vessel for a sample was replaced with material at a slightly different concentration, a correction was applied to account for the difference whenever necessary. In most cases the correction was negligible. The average surface area was calculated by estimating the amount of material that had dissolved up to the mid-point of the time interval and then calculating the relationship between the weight of the particles and their surface area. Most of the runs were made with crystalline materials that were more like parallelepipeds than like cubes or spheres. Therefore a relationship between surface area and volume was derived for such bodies on the assumption that each face of a crystal dissolved at the same rate per unit area. This leads to a rate of change in surface area which is less than the familiar two thirds power of the volume that is associated with equidimensional bodies.

In addition to the rate of solution measurements, such physical properties of the liquids as were unavailable in the literature (diffusivity, density, viscosity, solubility) were also determined.

RESULTS

The results are summarized in Figures 4 through 10. Detailed data are given in reference 2.

Baffled Vessels

Figure 4 shows the correlation for the baffled 6-in. vessel and the 2-in. turbine. The coordinates are defined, and the 95% confidence limits of the least-squares line through the data are indicated by the dashed curve above the line. (The lower confidence limit, which would be located a similar distance below the line, is not shown, to keep the figures uncluttered.) In subsequent plots this information is labeled in abbreviated form. The equations for the least-squares lines are also given in the figures.

Figure 5 shows the correlation for the baffled 12-in. vessel, including all the first-interval data for all the turbines and all the solids. Figure 6 compares the data for the three turbines and indicates the solids used in each run. Figures 7 and 8 are similar plots for the 18-in. vessel. Figure 9 is a plot of the data from the 30-in. vessel; the

Table 2. Ranges of Variables

Vessel size, in.	Turbine Diameter, in.	Speed, rev./min.	Liquid phase	Material	Solid Volume, %†	Particle sizes‡	Time between samples, sec.
6	2	578 to 1,440	Water	Boric acid	2.4	10/12, 18/20	25
		531 to 1,410	Sucrose*	Boric acid	1.65	8/10, 18/20	25, 35, 45, 60
12	3	589 to 1,206	Water	Boric acid	0.8, 2.4	12/14, 14/16, 16/20, 18/20	25
		610 to 1,170	Sucrose	Boric acid	1.63	12/14, 16/20	30, 60
		1,161	Brine	Salt	1.81	50-grain pellet	180
	Unbaffled 4	397 to 1,065	Water	Boric acid	0.8	16/20	25, 30
		422 to 1,106	Sucrose	Boric acid	3.3	16/20	30, 60
	4	333 to 865	Water	Boric acid	0.66, 0.8, 2.4, 4.9	10/12, 12/14, 14/16, 16/20, 18/20	25, 30
		368 to 621	Water	Benzoic acid	0.5, 9.3	0.1-g. pellet	45, 60, 420
		253 to 402	Sucrose	Benzoic acid	1.9, 9.3	0.1-g. pellet	240, 300, 420
		346 to 895	Sucrose	Boric acid	1.6, 3.3, 6.1, 23.2	10/12, 16/20	30, 60
	6	172 to 422	Water	Boric acid	0.8, 1.4, 1.5, 1.6, 2.4	6/8, 8/10, 16/18	25, 60
		171 to 177	Water	Benzoic acid	0.5, 9.3	0.1-g. pellet	60, 420
		170	Sucrose	Benzoic acid	1.9, 9.3	0.1-g. pellet	60, 420
		172 to 428	Sucrose	Boric acid	1.6, 6.1, 23.2	8/10, 12/14, 16/18	30, 60
		225 to 240	Brine	Rock salt	3.8	4/6	25, 117
18	6	196 to 677	Water	Boric acid	2.4	6/8, 14/16	25, 30, 60
		196 to 671	Sucrose	Boric acid	1.6	6/8, 8/10, 18/20	40, 60
		366 to 659	Brine	Rock salt	3.8	6/8	25, 40
	9	92 to 243	Water	Boric acid	2.4	8/10, 16/18, 18/20	25, 30, 60
		104 to 254	Sucrose	Boric acid	1.6	8/10, 16/18	60
		251 to 254	Brine	Salt	3.8, 4.4	4/6, 50-grain pellet	30, 60
	12	90 to 161	Water	Boric acid	2.4	10/12, 18/20	25, 30, 45, 60
		44 to 151	Sucrose	Boric acid	1.6	10/12, 18/20	60, 90, 120
		160	Brine	Salt	3.8, 4.4	4/6, 50-grain pellet	30, 60
	9	257 to 260	Brine	Rock salt	3.8	4/6, 6/8	30
30	12	141	Water	Boric acid	2.4	16/18	40
		145	Water	Benzoic acid	0.5	0.1-g. pellet	420
		136	Brine	Rock salt	3.6	6/8	30

* 45 wt. % sucrose in water.
† (initial volume of solids/volume of vessel) 100.
‡ 10/12 indicates through U.S. mesh 10, on 12; 50-grain pellets were 0.565-in. diam. by 0.531-in. long (over rounded ends); 0.1-g. pellets were 0.218-in. diam. by 0.126-in. long.

basis for the line through these data, which was obtained differently owing to the relatively few data points, is explained below. It should be noted that high values of k on the preceding plots are frequently those obtained from pellets, which had some tendency to disintegrate.

Figure 10 shows the least-squares lines for each vessel on the same plot. It is seen that the slopes are markedly similar and that, at a given Reynolds number, the larger the vessel the smaller the mass transfer coefficient.

The slopes of the lines for the 6-, 12-, and 18-in. vessels were plotted vs. the cube root of the volume of the vessel, which was chosen as the characteristic dimension because the height-to-diameter ratios were not exactly constant. Extrapolation of the least-squares line through these points indicated that the slope of the ln k vs. ln N_{Re} line for the 30-in. vessel should be 0.912. The lines for the 30-in. vessel shown on Figures 9 and 10 have this slope and are positioned so that the sum of the squares of the deviations of the points from the line is a minimum.

Table 3. Schmidt Numbers and Density Differences

System	Density difference, solid minus liquid, g./cc.	Schmidt number range, dimensionless
Benzoic acid–water	0.26	1,138 to 1,328
Benzoic acid–sucrose solution	0.06	62,000
Salt tablets–brine	0.71	826 to 989
Rock salt–brine	0.99	740 to 926
Boric acid–water	0.42	735 to 1,182
Boric acid–sucrose solution	0.23	27,400 to 55,000

A plot of the intercepts of the ln k vs. ln N_{Re} lines as a function of $V^{1/3}$ was used to yield least-squares equations relating the intercepts to $V^{1/3}$.

With the above approximations the experimental results for the baffled vessels may be expressed by

$$\ln(10\,k) = (I_1 \text{ or } I_2) + 0.85\,V^{0.02875}\ln(N_{Re}/10^4) \quad (3)$$

where

$I_2 = 0.8235 - 1.544\,V^{1/3} + 0.115\,V^{2/3}$, recommended for interpolations with vessels between 6 and 30 in.

$I_1 = 0.676 - 1.266\,V^{1/3}$, recommended for extrapolation to vessels less than 6 or more than 30 in.

The variance of estimate for all the first-interval data from Equation (3) is 0.0383, in units of $[\ln(10\,k)]^2$.

Unbaffled Vessels

The results for the unbaffled 12-in. vessel with the 4-in. turbine are summarized by

$$\ln(10\,k) = 0.747\ln(N_{Re}/10^4) - 0.671 \quad (4)$$

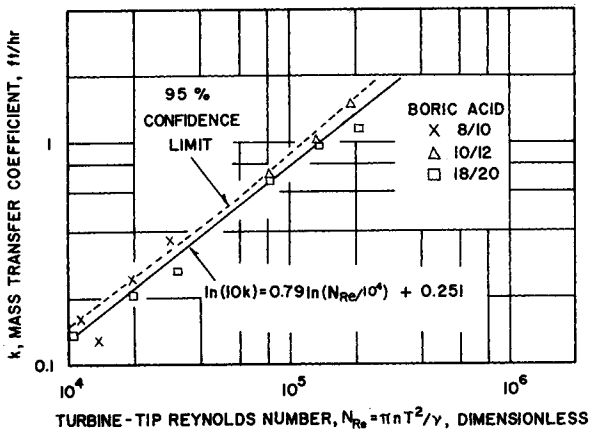


Fig. 4. Mass transfer coefficient vs. turbine-tip Reynolds number, 6-in. vessel, baffled, 2-in. turbine.

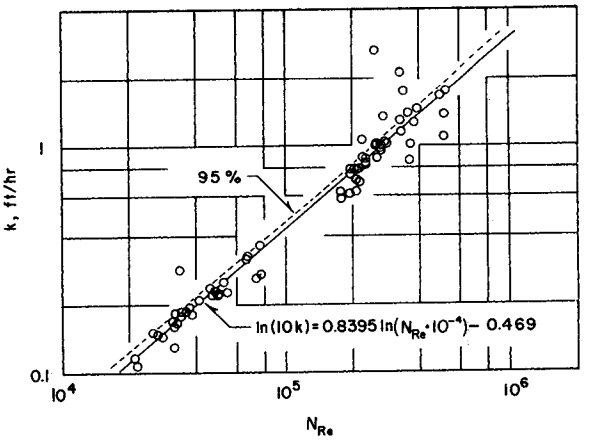


Fig. 5. k vs. N_{Re}, 12-in. vessel, baffled, 3-, 4-, 6-in. turbines.

The mass transfer coefficients in the unbaffled vessel are smaller than those in the baffled vessel, in the region under study.

EFFECTS ON k

All of the generalizations stated in this section refer to the baffled vessels.

Particle Size

The data indicate that there is no systematic variation of mass transfer coefficient with particle size. The 50-grain salt tablets, which were the largest particles used, exhibited high coefficients, but this is ascribed to a tendency for these tablets to disintegrate rather than to any fundamental effect of particle size.

Density Difference

The difference in density between solid and liquid does not appear to influence the mass transfer coefficient.

Schmidt Number

Schmidt number, despite the large range studied, does not appear to influence the mass transfer coefficient to any great extent. Since kinematic viscosity appears in both the Reynolds and the Schmidt numbers, when the Schmidt number is high the Reynolds number tends to be low and vice versa. Therefore it is difficult to obtain a wide range of Schmidt numbers at a given value of Reynolds number. This is especially true when either suspension of particles or prevention of emulsification are auxiliary requirements. Therefore it seems unnecessary to include both N_{Sc} and N_{Re} to account for the variation in k, since N_{Re} alone appears adequate.

Ratio of Turbine Diameter to Tank Diameter

The ratio of turbine diameter to tank diameter exerts a negligible influence on the mass transfer coefficient over a wide range of the ratio (0.27 to 0.54 in the 12-in. vessel, 0.34 to 0.7 in the 18-in. vessel).

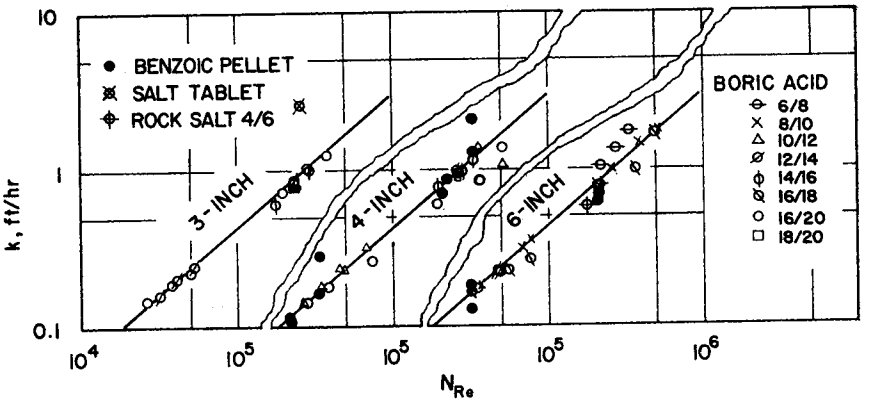


Fig. 6. k vs. N_{Re}, 12-in vessel, baffled, comparison of turbines.

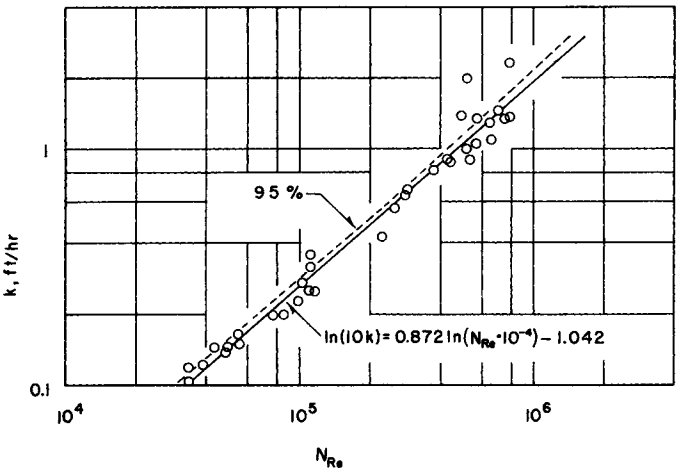


Fig. 7. k vs. N_{Re}, 18-in. vessel, baffled, 6-, 9-, 12-in. turbines.

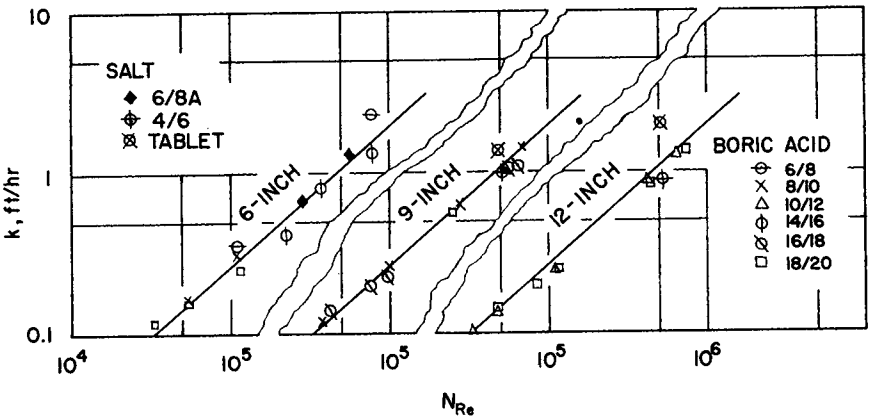


Fig. 8. k vs. N_{Re}, 18-in vessel, baffled, comparison of turbines.

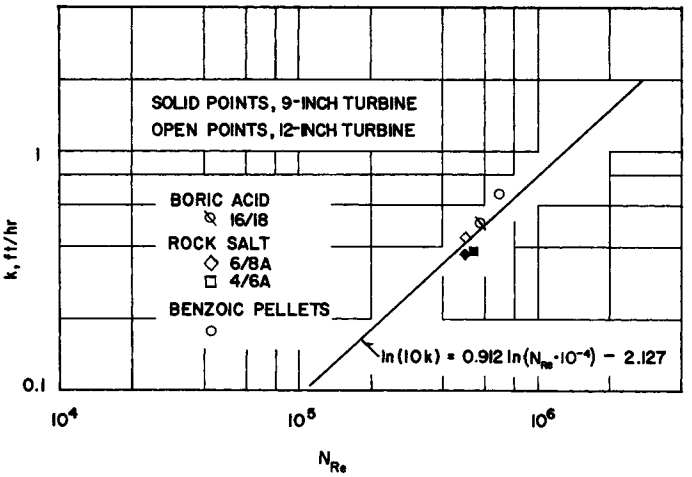


Fig. 9. k vs. N_{Re}, 30-in. vessel, baffled, 9-, 12-in. turbines.

Reynolds Number

The data indicate that k increases with the Reynolds number raised to a power between about 0.8 and 0.9, with the larger vessels favoring the larger exponents.

Vessel Size

Larger vessels give smaller mass transfer coefficients at the same Reynolds number.

Inventory of Solids

There is a tendency toward somewhat larger coefficients when the inventory of solids is increased, with other conditions held constant. However this is believed to be caused by a greater relative proportion of fines in those runs with the large inventories. This could be caused by the greater difficulty of removing fines from the larger inventory before it was added to the vessel, or by the increased collision frequency, and consequently greater attrition rate, in the larger inventory cases. In any event the effect of inventory is not large.

Power per Unit Volume

Rushton (*17*) has shown that for Reynolds numbers greater than about 10^4 the power number in open, baffled vessels equipped with six bladed turbines is constant. Laity and Treybal (*10*) have demonstrated that this is likewise true for similar vessels when run covered and full of liquid. Under these circumstances, with all other things the same, the power per unit volume in two different vessels may be expressed as

$$\frac{(P/V)_1}{(P/V)_2} = \frac{V_2}{V_1}\left(\frac{N_{Re1}}{N_{Re2}}\right)^3 \frac{T_2}{T_1} \qquad (5)$$

Table 4 compares the power per unit volume for the four vessels at $k = 0.5$ ft./hr. Since the slopes of the curves of k vs. N_{Re} are all about the same, the comparison would be approximately the same at other values of k.

Power per unit volume apparently exhibits a maximum with vessel size.

When applied to the same vessel with turbines of different sizes, Equation (5) indicates that, for the same power input, a higher Reynolds number is obtained with a larger turbine. Since k increases with N_{Re} independent of turbine size, a larger turbine will effect a given amount of mass transfer for a smaller expenditure of power. Whether this will also correspond to a more economical operation depends on many other factors that in turn depend on the specific application under consideration.

Table 4. Comparison of Power per Unit Volume in Various Vessels at the Same Value of

Vessel size, in.	Ratio: Power per unit volume in 6-in. vessel to power per unit volume in other vessel
6	1
12	1.73
18	1.78
30	0.62

Table 5. Mass Transfer Coefficients Predicted by Aksel'rud's Equation

Particle diameter, in.	k, ft./hr. for boric acid in Water	Sugar solution
0.02	3.82	0.424
0.20	2.15	0.238

Conventional Treatment of Data

The new data are also amenable to treatment through the use of the customary dimensionless groups. The slopes of the lines of Figure 10 are nearly equal, at an average value of 0.833, and at a given N_{Re} the product kd is practically constant for all vessels. Here d is the diameter of a cylinder of equal height and diameter and of the same volume as the vessel. Therefore kd is proportional to $(N_{Re})^{0.833}$. When $(kd/D)/(N_{Re})^{0.833}$ is plotted against N_{Sc} on logarithmic paper, the points can be represented by a straight line whose slope might be anywhere between 0.45 and 0.55, owing to the scatter of the data. A value of 0.5 was chosen, so that the correlation of Figure 11 results. The dimensionless-group equations which describe all the data reasonably well are then

$$N_{Sh} = 0.02\,(N_{Re})^{0.833}\,(N_{Sc})^{0.5} \qquad (6)$$

or, in terms of the more conventional Reynolds number $N_{Re}' = T^2n/\nu$,

$$N_{Sh} = 0.052\,(N_{Re}')^{0.833}\,(N_{Sc})^{0.5} \qquad (7)$$

The dependence of the mass transfer rates on diffusivity which these equations imply is only apparent. The ratio of the group $D\mu$ to temperature, which has long been used to correlate diffusivities, indicates that D varies as $1/\mu$. Since the densities of common liquids

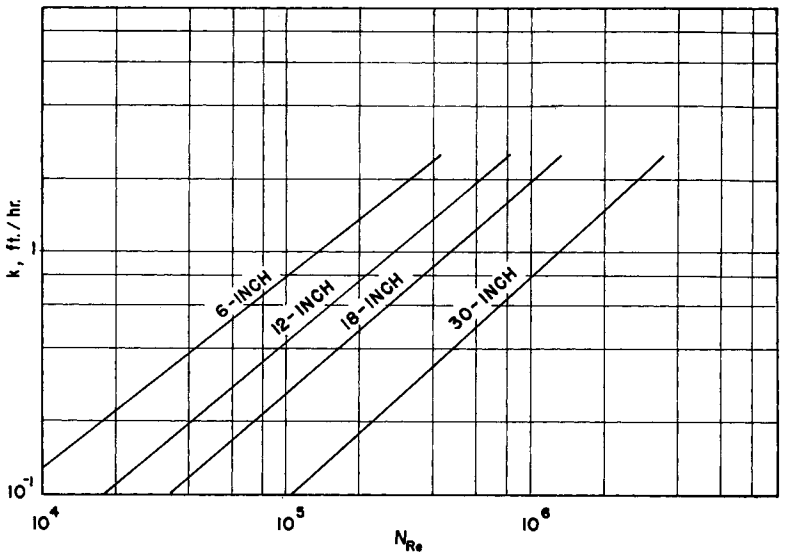


Fig. 10. k vs. N_{Re}, all vessels, baffled.

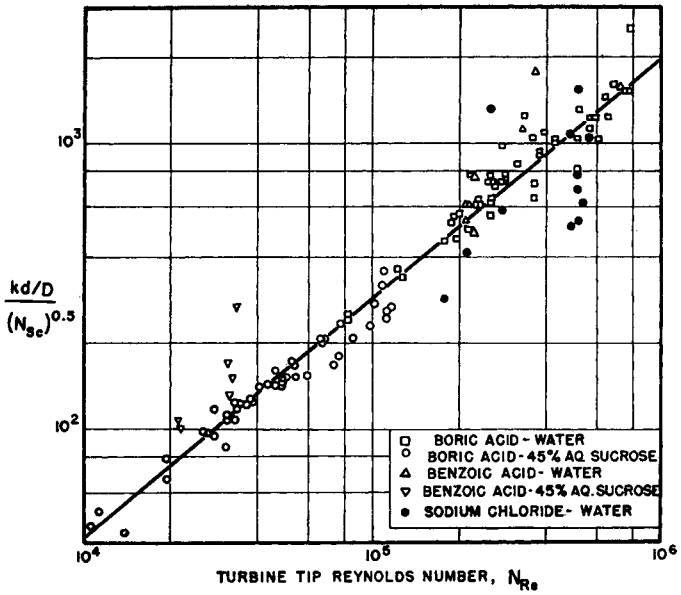


Fig. 11. Dimensionless correlation for baffled vessels.

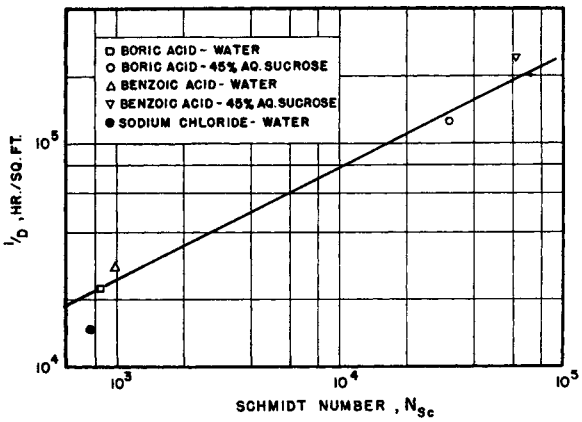


Fig. 12. Relation between diffusivities and Schmidt numbers, 25°C. The line has a slope 0.5.

lie within a narrow range, it follows that $1/D$, which is contained in N_{Sh}, can be expected to relate to $(N_{Sc})^{0.5}$. This is the case for the large range of Schmidt numbers employed here, as demonstrated in Figure 12, where the data are reasonably well described by the straight line of slope 0.5. It follows that $1/D$ and $(N_{Sc})^{0.5}$ essentially cancel from Equations (6) and (7) and that Figure 11 is practically a plot of kd against N_{Re}. The empirical equation developed earlier more correctly reflects the observed role of diffusivity.

COMPARISONS WITH DATA OF OTHERS

Mass Transfer Coefficients

Hixson and Baum (3) worked in unbaffled vessels with propeller agitators. The mass transfer coefficients for one of the systems of this study predicted by their equation were compared with the unbaffled data obtained in the present work. Their equation predicts a k about 1.9 times that observed for the sugar solutions ($N_{Sc} \approx 28{,}000$) and about 1.3 times that observed for the aqueous solutions ($N_{Sc} \approx 830$). Their agitators were located closer to the bottom of the vessel and acted to lift the particles through the propeller, which could promote better mass transfer in an unbaffled tank.

Aksel'rud's (1) equation has been used to estimate the mass transfer coefficients to be expected with boric acid in the experimental range of interest. The results are shown in Table 5.

Despite the obvious apparent shortcomings of Aksel'rud's equation, the magnitudes of the predicted values of the mass transfer coefficient are correct.

Wilhelm, Conklin, and Sauer (21) found the coefficients shown in Table 6 when dissolving rock salt in water, using a three bladed propeller at a constant speed in an unbaffled crock. Equation (4) predicts a k of 0.9 ft./hr. for these conditions, ignoring the differences in geometry, which is a surprisingly good check.

Turbine Speed

One of the experiments by Nagata, Adachi, and Yamaguchi (14), using a four bladed paddle close to the bottom and benzoic acid pellets of 0.04 g. each dissolving in caustic solution, was done with 15% baffles in a 0.8-liter vessel. They found reciprocal neutralization time proportional to $n^{0.572}$ in the region of full suspension, and their unbaffled data show less dependence on speed.

Humphrey and Van Ness (6) found the mass transfer coefficient varied as the 0.87 power of the speed of rotation.

Suspension Point

Zwietering (22) presents a correlation for the amount of mixing required for full suspension of solid particles in tanks of various sizes, based on experiments using salt and sand in various liquids and with various types of agitators. In the case of each run with salt in the study reported here a comparison was made between the agitator speed predicted by Zwietering's correlation and the actual speed used. The ratio of actual speed to that required by Zwietering's correlation ranges from 0.48 to 1.18. The mass transfer coefficients for those runs where the ratio is low, however, fit the correlation of Equation (3) equally as well as those for which the ratio is high. It is therefore concluded that if Zwietering's criterion is satisfied, suspension is full as far as mass transfer is concerned. But even if Zwietering's criterion is not satisfied, suspension may still be adequate for mass transfer purposes. In other words Zwietering's criterion predicts a point of adequate agitation for mass transfer but not the point of inadequate agitation.

Table 6. Coefficients Found by Wilhelm, Conklin, and Sauer*

Particle size, in.	k, ft./hr.
0.0748	0.984
0.13	1.023
0.173	1.122
0.232	1.181
0.303	1.338

*Initial charge of rock salt 40% of that required to saturate the 10 liters of water used at 15°C.

COMMENTS ON TURBULENCE

The motion inside the baffled vessels was definitely turbulent. It is therefore not so surprising that the mass transfer coefficient turns out to be independent of Schmidt number, or molecular diffusivity. At the present time, however, not enough is known about the details of turbulent motion in baffled vessels to be able to predict the mass transfer coefficients from data on other properties of the fluid motion. In fact the problem may be turned around, and the mass transfer coefficients may be used as a measure of the average level of turbulence in the vessels.

The general equation for turbulent transfer may be expressed as

$$Q = -v\,l'\frac{d\theta}{dh} = -E\frac{d\theta}{dh} \qquad (8)$$

In mass transfer θ becomes the concentration, and the flux of mass may be expressed as

$$N = -E\frac{dc}{dh} = E\left(\frac{c_s - c}{l'}\right) = k\,(c_s - c) \qquad (9)$$

$$\therefore k = \frac{E}{l'} = v \qquad (10)$$

This expression indicates that the turbulent mass transfer coefficient is equal to the mean fluctuating velocity. Anything that promotes fluctuations in velocity should promote mass transfer.

It has been observed that the mass transfer coefficient in unbaffled vessels is less than that in baffled vessels, with other conditions equal. The pattern of motion in an unbaffled vessel, which consists of a rotational swirl superimposed on the turbine pattern, allows packets or eddies of fluid to take long flights between collisions with objects that promote velocity fluctuations. Therefore it is not unreasonable that the mass transfer coefficient should be smaller. Under conditions when the turbine speed is insufficient to lift the particles, on the other hand, others have observed that the coefficient in unbaffled vessels is greater than that in baffled vessels. (In other words the slopes of k vs. N_{Re} persist to low N_{Re}, and eventually k unbaffled $> k$ baffled.) A possible explanation for this may be that the swirling velocity in the unbaffled vessels, which can build up to appreciable magnitudes, by tumbling the particles on the vessel bottom actually exposes more surface and provides a greater relative velocity between solid and liquid than is produced in the baffled vessels.

It should be noted that it is apparently not sufficient to have eddy diffusivity large with respect to molecular diffusivity for the latter to be negligible in mass transfer. According to Equations (9) and (10), when the film concept of molecular diffusion is recalled, one sees that it is necessary to have

$$\frac{E}{l'} >> \frac{D}{B} \qquad (11)$$

before molecular diffusion becomes negligible. This is a more stringent condition than $E >> D$, since certainly $l' >> B$.

It is well known that in pipes $(E/l') \sim (N_{Re})^{0.9 \text{ to } 1}$, with the exponent on the Reynolds number tending toward unity as the roughness of the pipe increases. The present results indicate that $k = E/l' \sim (N_{Re})^{0.8 \text{ to } 0.9}$. If $l' \sim (N_{Re})^{0.1}$ in the baffled tanks, then the eddy diffusivity in pipes and in baffled vessels is similar. This would imply that l' increases as N_{Re} increases, which makes the condition expressed by Equation (11) even more stringent.

An attempt was made to compare the eddy diffusivities obtained in fluidized beds with those obtained in this work, without too much success because of fundamental differences. Eddy diffusivity goes through a maximum as the velocity increases and the bed expands from the packed condition to a highly dilute state. A plausible explanation of this behavior may be as follows. When the velocity is low, the distance between particles is small, and since $E \sim N_{Re}$, E is small; as the bed expands, N_{Re} increases, and therefore E also increases; eventually E becomes independent of the distance between particles and approaches that characteristic of the pipe. The picture is complicated because the particles themselves generate turbulence in the fluid, accounting for the maximum in the curve of E vs. velocity, rather than a monotonic increase toward the diffusivity characteristic of the pipe.

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NOTATION

a = a constant, dimensionless
$\overline{A}$ = average surface area of particles, sq. ft.
b = slope of least squares line of ln k vs. ln N_{Re}
B = film thickness, ft.
c = concentration, moles/unit volume
c_s = solubility, moles/unit volume
d = differential
d = diameter of vessel, ft.
D = molecular diffusivity, sq. ft./hr.
E = eddy diffusivity, sq. ft./hr.
h = distance, ft.
I = intercept of least-squares line on plot of ln$(10\,k)$ vs. ln$(N_{Re}/10^4)$, k in ft./hr.
I_1 = first-order approximation to I
I_2 = second-order approximation to I
k = mass transfer coefficient, ft./hr.
l' = mixture length, ft.
n = speed of rotation of turbine, rev./hr.
N = flux of mass, moles/hr., sq. ft.
P = power for agitation, hp.
Q = mean rate of transfer of a property per unit time per unit area
N_{Re} = Reynolds number, $n\pi T^2/\nu$, dimensionless, $N_{Re}' = N_{Re}/\pi$
N_{Sc} = Schmidt number, ν/D, dimensionless
N_{Sh} = Sherwood number, kd/D, dimensionless
t = time, hr.
T = turbine diameter, ft.
V = volume of vessel, cu. ft.

Greek Letters

μ = viscosity, lb./(ft.)(hr.)
ν = kinematic viscosity, sq. ft./hr.
v = root-mean-square velocity fluctuation, ft./hr.
ρ = density of liquid, lb./cu. ft.
ρ_s = density of solid, lb./cu. ft.
$\Delta\rho$ = $(\rho_s - \rho)$ = difference in density, solid minus liquid, lb./cu. ft.
θ = property
π = 3.14159

Subscript

i = beginning of an interval

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